

A possible resonance mechanism of earthquakes

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Abstract

It had been observed by Linkov, Petrova and Osipov (1992) that there exist periodic 4-6 hours pulses of $\sim 200 \mu\text{Hz}$ seismogravitational oscillations (SGO) before 95 % of powerful earthquakes. We explain this by beating between an oscillation eigenmode of a whole tectonic plate and a local eigenmode of an active zone. The beating transfers the oscillation energy from the remote zone of the tectonic plate to the active zone, triggering the earthquake. Oscillation frequencies of the plate and ones of the active zone are tuned to a resonance by an additional compression applied to the active zone due to collision of neighboring plates or the magma flow in the liquid underlay of the asthenosphere (the upper mantle). In the case when there are three or more SGO with incommensurable difference frequencies $\omega_m - \omega_n$ the SGO beating pattern looks quasi-random, thus masking the non-random nature of the beating process. Nevertheless, we are able to discuss a possibility of the short term earthquakes predictions based on an accurate monitoring of the beating dynamics.

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I. QUALITATIVE PICTURE

Potential energy of an elastic object is usually defined by an appropriate quadratic form. Enhancing of the quadratic form implies increasing of the eigenfrequencies, recall the stretching of a guitar string or bending of a relatively thin plate caused by a normal force. Monitoring of frequencies of SGO in connection with elastic energy storage on the tectonic plate (in particular, due to bending under a normal force) was discussed in (Petrova and Pavlov 2008). On the contrary, a tangential compression of an elastic plate implies decreasing of the eigenfrequencies, see (Heisin 1967). While the lower eigenfrequencies of a small tectonic plate lie much higher than the lower eigenfrequencies of a similar large plate, the tangential compression may bring the lower eigenfrequency of the small plate in resonance with some eigenfrequencies of a large plate. Therefore, the oscillation energy of the corresponding mode on the large tectonic plate may be transferred to the small plate in contact (or just to an appropriate compressed area - an active zone) thanks to the beating phenomenon, which happens between two coupled oscillators with close frequencies. For instance, the dynamics of two coupled oscillators with almost equal eigenmodes frequencies $\bar{\omega} \pm \omega_\delta$, with coordinates x and y and interaction δxy , is described in (Landau and Lifshitz 1969), as

$$x = A \cos(\omega_\delta t + \varphi) \cos(\bar{\omega} t), \quad y = B \sin(\omega_\delta t + \varphi) \cos(\bar{\omega} t).$$

Here the beating frequency $\omega_\delta \ll \bar{\omega}$. This example illustrates periodic (with the frequency ω_δ) migration of energy from one plate to another. General algebra of beating is reviewed in Appendix B. Further calculations concerning the resonance interaction of SGO modes for contacting small and large plates requires deeper mathematical analysis and will be published by V. Flambaum, G. Martin and B. Pavlov in a separate paper “On the resonance interaction of seismogravitational modes on tectonic plates” (in preparation). Here we present simple estimates. The frequency of a transverse plane wave on a large thin plate depends on the pressure as in (Heisin 1967).

$$\omega = 2\pi \nu = k \sqrt{\frac{Dk^2 - Q}{\rho h}}, \quad (1)$$

where Q is the compressing tangential force per unit length applied along the direction of the motion of the wave, ρ is the density and h is the thickness of the plate, $D = \frac{h^3 E}{12(1-\sigma^2)}$, E is the Young’s modulus, and σ is the Poisson ratio. Solution for a spherical wave is

presented in appendix C. We can use Eq. 1 to make a simple estimate of the effect of the compressing tension. For the fundamental mode on a finite plate the length of the wave vector is estimated as $k \sim 1/L$, where L is the size of the plate. Hereafter, for a typical tectonic plate we assume $E = 17.28 \times 10^{10} \text{ kg m}^{-1}\text{s}^{-2}$, $\rho = 3380 \text{ kg m}^{-3}$, $\sigma = 0.28$, and a typical frequency of a large tectonic plate vibration $\nu \sim 170 - 190 \mu \text{ Hz}$, see (Petrova and Pavlov 2008). This allows to estimate the tangential tension required to bring lower eigenfrequencies of a small plate into resonance with lower eigenfrequencies of the large plate. Significant reduction of the frequency of a small plate (which brings it to a resonance with a frequency of a large plate) happens near the root of the expression under the square root in Eq. (1), $Dk^2 = Q$. This gives us an estimate for the required pressure:

$$P \sim 10^{10} \left(\frac{h^2}{L^2} \right) \text{ Pa} \quad (2)$$

Hereafter we assume that the active zone is a small plate Ω which is in contact with a large plate Ω_t . The typical linear size of the small plate is $L \sim 100 - 200 \text{ km}$. Large tectonic plates extend to $L_t \sim 1000-10000 \text{ km}$ and are $h \sim 200 - 300 \text{ km}$ thick on the continents, but much thinner, $h = 30 - 100 \text{ km}$ on the oceans bottoms. A small ratio h/L may reduce the required compression for the resonance up to two orders of magnitude. An upper estimate of an existing compression may be given by a crushing pressure $P \sim 10^9 \text{ Pa}$ of the material composing the plates.

The resonance may also appear for a much smaller compression, if the tectonic plate is thick and the small one (active zone) is relatively thin (e.g. under ocean). Indeed, for small Q the plate frequency is

$$\omega \sim (1/L^2) \sqrt{(D/\rho h)} \propto h/L^2, \quad L^2/h \sim L_t^2/(h_t n^2)$$

There may be also resonances between the fundamental frequency of the small plate and the higher SGO modes of the large tectonic plate ($kL_t \sim n$, $n = 1, 2, \dots$). For small Q the tectonic plate frequency and the resonance condition are reduced to

$$\omega \sim (n^2/L_t^2) \sqrt{(D/\rho h_t)}, \quad L^2/h \sim L_t^2/(h_t n^2).$$

Finally, there may be a resonance between the different types of the modes on different plates which have different fundamental frequencies (e.g. transverse, longitudinal and surface (Rayleigh) modes). The lowest mode on the small plate may resonate with a higher mode on

the large plate. The increasing compression on the small plate in this case defines scanning of the frequency until it comes into a resonance with one of the frequencies on the large plate. An efficient mechanism of the scanning may arise also due to a bending of the plates by the non-tangential force. The bending increases the potential energy of the elastic deformation and the oscillation frequencies. The change of frequencies of the tectonic plates due to the bending has been discussed, e.g. in (Petrova and Pavlov 2008). Moreover, this effect is seen in the data of observations presented below, see Fig. 1.

Assume that initially the SGO are registered on a large plate. If the small and large plates are disconnected, their oscillations are independent. The compression may bring the frequency of the small plate in resonance with some frequency of SGO of the large plate. Then even a weak interaction between the SGO processes on the plates results in forming the perturbed SGO mode of the pair of plates $\Omega \cup \Omega_t$, manifesting the beating of the modes and causing migration of energy from one plate to another. The sum of energies of oscillations of the plates remains constant. In absence of exact resonance the energy transfer is not complete, but when the resonance becomes sharper, the energy transfer becomes fuller. When the energy of the large plate comes to the small plate, the mean amplitude A of SGO on the small plate becomes enormous due to the energy conservation law $A/A_t \sim L_t\sqrt{h_t}/L\sqrt{h}$, triggering the earthquake.

Even a partial transition of energy of a resonance SGO -mode from the large tectonic plate to the active zone may cause an enormous effect. For instance the elastic energy stored in a single SGO mode with frequency $200 \mu\text{HHz}$ and amplitude $2 \times 10^{-3} \text{ m}$ on the tectonic plate with area 10^{14} m^2 , thickness 10^5 m and density 3380 kg m^{-3} is estimated as 54×10^9 joules, which is almost equivalent to the 4M earthquake in Johannesburg (South Africa) November 18, 2013. Even a small part of this amount may trigger a powerful earthquake, and should be taken into account when considering a realistic mechanism of the earthquake. Estimation of the transfer coefficient defining the transfer of energy in course of resonance beating of SGO modes would probably help to develop more realistic Earthquake theoretical scenario.

The above resonance interpretation of the earthquake mechanism may be useful for short-term earthquake predictions, see section 3, Conclusion. For instance, if there are only two interacting modes with frequencies ω, ω_t , the beating is periodic. If we have registered two pulses manifesting the moments when the migrating energy is accumulated on the large

plate, then in a half period after the second pulse the whole energy will be already on the small plate, and may trigger the earthquake. But even if we observed a single pulse and registered the moments of maximal and the previous moment of minimal energy (amplitude) in the remote zone of the large plate, we are able to predict the moment when the energy is maximal on the small plate. This moment is coincident with the next minimum of the energy on the large plate. Note that the observations may be done in the remote zone of the large plate, very far from the active zone. The case of perturbation of a multiple eigenfrequency may be considered based on an appropriate aperiodic beating algebra, see Conclusion, section 3.

Suggestion of experiment. Mathematics can't yet provide reliable results for resonating eigenmodes of plates under various (tangential or/and normal) tensions, beating frequencies and transferred energy. However, a more reliable way to investigate these problems may be a laboratory experiment with plates of different sizes and shapes. For instance, most natural question is one on existence and structures of oscillation modes, localized essentially on a small active zone Ω or on the complement Ω_t , with close frequencies $\omega \sim \omega_t$, yet without the resonance condition imposed, $\omega \neq \omega_t$. For frequencies approaching the resonance $\omega - \omega_t \rightarrow 0$, the experiment may help to recover their dependence on tensions applied on the active zone, and give essential data for constructing and fitting of the mathematical model of the general (aperiodic) scenario of beating of the perturbed multiple modes.

The whip effect. The beating phenomenon is observed while the beating frequency is sufficiently large compared with the speed of change the frequencies, caused by the bending of the large plate . If the changes go faster, then possibly the resonance condition is satisfied only once during the observation period. Nevertheless the earthquake may be triggered in this case too, by an analog of the celebrated whip effect, which manifests, due to the energy conservation, the amplitude growth while the wave, running along the thinning channel, is approaching the thin end (of the whip). Indeed, that may happen on the ocean bottom, if the tectonic plate is gradually thinning along the wave track. Generally a combination of the beating and whip scenarios of the earthquake is possible, while few beating are terminated by the whip effect triggering the earthquake at the moment of reaching resonance.

In this text we neglect an important phenomenon of dissipation in the SGO process. A preliminary discussion of this matter may be found in (Ivlev et al. 2012).

II. OBSERVATIONS AND THEIR RESONANCE INTERPRETATION

In the paper by Linkov et al (1992) the pulses of SGO were discussed as typical precursors of powerful earthquakes, arising with probability 95 %. The spectral nature of the SGO was demonstrated in (Petrova and Pavlov 2008). Additional unpublished information was kindly provided to us by L. Petrova, who provided us so-called spectral-time cards constructed by herself based on monitoring of the SGO process preceding the earthquake 26 September 2005 in Peru (see Fig.1). L. Petrova also attracted our attention to some details on the cards, which may be considered as precursors of the earthquake, but were not interpreted yet properly. First, there are two “pulses” registered on SSB station (France) in the zones $\Delta_2^{SSB} = (190, 200) \times (55, 65)$; $\Delta_3^{SSB} = (200, 210) \times (145, 165)$ introduced in the Appendix A below, separated by the time interval 96 hours. We believe now, based on above resonance interpretation, that they are SGO -beating on the large plate situated on the way of the waves coming from Peru location to the SSB station in France. Secondly, she noticed a “shock” between the pulses at $T = 87$ h, causing generation of three oscillation modes, clearly registered on the INU-station (Japan). We guess, again based on our resonance interpretation, that the corresponding signal may come from the small plate. Finally, the earthquake succeeded at the moment $T = 172$ h, in 48 hours after the second pulse, and 96 hours after the shock at the moment 87 h, see details in Appendix A. Notice that our guess does not permit to describe in detail the way of migration of energy from the active zone in Peru to the remote zone on the large plate Ω_t , where the SSB station is located. But this is typical for most of experiments with resonance systems, where just “some” interaction of oscillators involved is important.

The spectral-time cards also give us an evidence of the bending effect which may tune frequencies of the plates to the resonance. Firstly, we can extract it from presence, in the remote zone, of modes with growing (for growing bending) and decreasing (for relaxed bending) frequencies. Indeed, they are easily seen on the spectral - time cards as ridges extending from the left down to right up or - vice versa - from left up to right down. We may guess that these ridges arose from the unperturbed modes on the big plates Ω_t , which are excited, on the perturbed background, by the shock at the moment $T = 87$, and affected by the bending or relaxation (due to partial destruction) of the large plate under the *normal tension*. Secondly , we guess that there exist also an unperturbed mode on the large plate,

with an almost resonance frequency, which has a “knot” at the location of the active zone, so that it’s frequency ω_t is not affected by the local bending at the location. We guess that it stays in resonance with the corresponding mode on the active zone $\omega_t - \omega \approx 0$, tuned by the *tangential tension*. We guess that it forms the beating pattern, registered on the domains $\Delta_2^{SSB} = (190, 200) \times (55, 65)$; $\Delta_3^{SSB} = (200, 210) \times (145, 165)$ of the SSB card. A minor shift “up” of the resonance frequency $(190, 200) \rightarrow (200, 210)$ may be caused by the details of the shape of the contact between the plates, which do not provide an exact separation of compressing and normal forces on the contact.

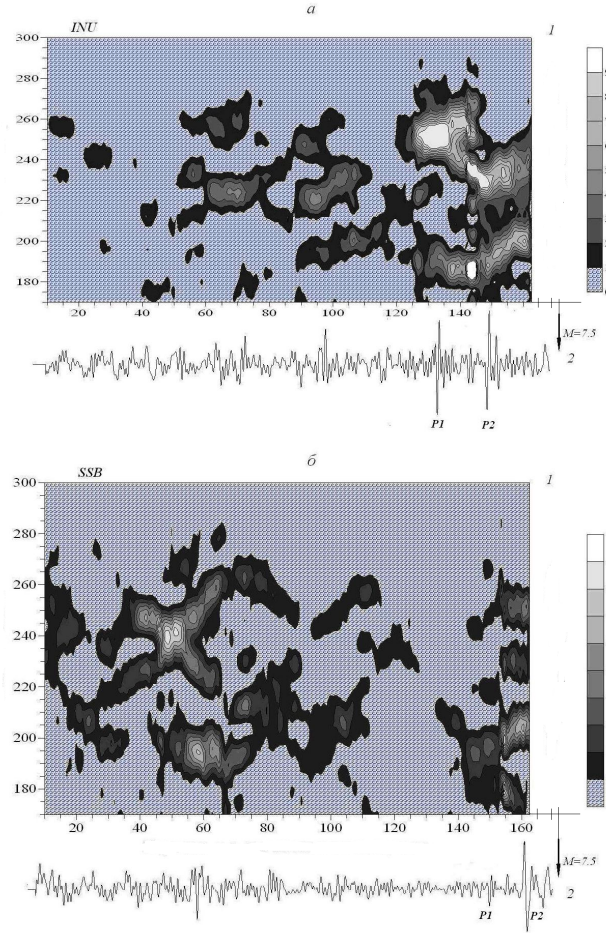


FIG. 1: Spectral-Time cards constructed by L. Petrova based on seismo-gravitational oscillations recorded on INU and SSB stations, in Japan and in France respectively, during the period 18-26 September 2005 preceding the strong Earthquake in North Peru. The horizontal axis for time, is graded in hours, the vertical axis, for the frequencies, is graded in μHz .

III. CONCLUSION

The M8 Global test algorithm of the earthquake prediction, has been designed in 1984 at the International Institute of Earthquake Prediction and Mathematical Geophysics (Moscow) based on the observation that almost 80 % of actual events at the selected location arise due to the stress built up thanks to previous events at the corresponding Earthquake-prone (active) zone.

Though the algorithm was extremely efficient providing higher than 99 % confidence level for defined Time intervals of Increased Probability (TIP) of the Earthquakes, yet some of highly dangerous events, like Tohoku earthquake in Japan on March 11 2011 were not predicted, because the black box constructed based on the M8 Global Test algorithm , removed the warning 70 days before the earthquake, see the retrospective analysis of the Global Test effectiveness in (Kossobokov 2013).

We did not plan, in our note, “to come out with exact short term prediction algorithm” but, inspired by the author of the above review, wish to provide our vision of the problem from the viewpoint of mechanics and spectral theory. We hope that our observations may attract attention of experimentalists to the resonance phenomena (beating) in the SGO process on elastic thin plates.

We keep in mind that , in the linear approximation of the elasticity theory, the seismo-gravitational oscillations with the frequency circa $200 \mu\text{Hz}$ and the corresponding pulsations as precursors of the earthquakes, correspond to very long transverse waves. This long waves are not affected by local shape variations of the plates (coarse grain or fine grain) and the relief. This allows to model the SGO in the laboratory, clarifying the basic questions of the SGO process, including the beating , the energy migration and triggering an earthquake. This may help further refining of the M8 Global Test algorithm, with regard of possible resonance effects and the energy migration, to improve the TIP prediction results.

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IV. APPENDIX A: MORE ABOUT SPECTRAL-TIME CARDS

The spectral - time cards are obtained from the seismograms via averaging of the square amplitudes of the oscillations with certain frequency on the systems of 20 hours time - windows, selected by shifting an initial window by 30 minutes on each step. The seismograms underwent previously a double filtration with Potter filter: the high frequency one, with boundary period 6 h, and one more, with the window 1-6 h. The boundaries of the domains on the cards, where the averaged amplitude of the SGO mode with certain frequency ν , at the given moment T of time, exceeds given value A , form a system of isolines in the frequency/time coordinates ν, T . The relief of the window averaged squared amplitude on the cards is graded by the isolines, with the step $\delta A^2 = \frac{1}{10} [A_{max}^2 - A_{min}^2]$, and is painted accordingly between the isolines, with dull grey for the background value A_{min}^2 of the squared amplitude and white for the maximal value A_{max}^2 . The results of monitoring represented on the cards correspond to the trains of SGO with growing *a*), constant *b*) and decreasing *c*) frequency ν

a) INU 88 \rightarrow 107, $220 < \nu < 235$,

b) SSB 50 \rightarrow 65, $\nu \approx 195$,

c) SSB 70 \rightarrow 87, $265 > \nu > 247$,

and brief (6-20 hours) stationary SGO modes with high amplitudes in the (conventionally) rectangular zones on the cards in frequency - time coordinates as $\Delta\nu, \Delta t \Delta_1^{SSB} = (235, 245) \times (45, 55)$

$$\Delta_2^{SSB} = (190, 200) \times (55, 65)$$

$$\Delta_3^{SSB} = (200, 210) \times (145, 165)$$

$$\Delta_1^{INU} = (240, 260) \times (125, 139)$$

$$\Delta_2^{INU} = (225, 224) \times (142, 150)$$

$$\Delta_3^{INU} = (180, 190) \times (142, 145)$$

One can see on the SSB card two groups of stationary modes with almost equal frequencies and visually similar relief in Δ_2^{SSB} and Δ_3^{SSB} , which were identified by L. Petrova as “seismo-gravitational pulsations”, see comments in previous section. Dr. Petrova also attracted our attention to a family of prolonged (up to 50 hours) SGO trains with growing frequency (1 $\mu\text{Hz}/\text{hour}$) on the intervals $(50, 160)^{INU}$ $(0, 110)^{SSB}$ and almost total absence of the modes with growing frequency on the complementary intervals $(0, 50)^{INU}$, $(110, 145)^{SSB}$.

Vice versa, the modes with decreasing frequency are absent on the time interval $(87, 120)$ on both cards. Dr. Petrova suggested that probably some important event (a shock?) succeeded at the moment 87, which excited three SGO modes on the interval $(87, 120)^{SSB}$, two of them also clearly seen on $(87, 120)^{INU}$, see our interpretation in previous section. The extent of the clearly seen part of the middle train, measured on the middle line of the corresponding “ridge” on the interval $(87, 120)^{SSB}$ is about 24 hours, and the extents of the upper and lower modes are longer and shorter than the middle one by the intervals proportional to the difference of frequencies of the modes.

V. APPENDIX B: ALGEBRA OF BEATINGS

The problem on beating of the seismogravitational modes has a simple algebraic nature: it is modeled by a system of coupled oscillators with multiple eigenfrequency p_0 which is perturbed such that the multiple eigenvalue is split into a starlet $p_\delta^s = p_0 + \delta\alpha^s$ under a minor perturbation, which also transforms the initial eigenbasis $\{e_0^s\} \rightarrow \{e_\delta^s\} \equiv U_\delta \{e_0^s\}$, with an unitary generator which, in simplest case, is represented by an exponent of an antihermitian matrix $U_\delta \equiv \exp[\delta B]$. Both the starlet and the generator of rotation the basis are usually found in terms of normal modes, and the perturbed evolution is represented as a linear combination of the normal modes

$$u(t) = \sum_s A_\delta^s \cos[(p_0 + \delta\alpha^s)t + \varphi_s] U_\delta e_0^s$$

The energy $E_\delta(u) = \frac{1}{2} [\|u_t\|^2 + \sum_s |A_\delta^s|^2 |p_\delta^s|^2]$ of the perturbed evolution $u_{tt} + A_\delta u = 0$ and the unperturbed energy of the unperturbed evolution $u_{tt} + A_0 u = 0$ $E_0(u) = \frac{1}{2} [\|u_t\|^2 + |p_0|^2 \|u\|^2]$ are conserved. The unperturbed values of energy $E_0(P_0^s u)$ of the projections $P_0^s u(t) \equiv e_0^s \langle e_0^s, u(t) \rangle$ of the perturbed evolution onto the eigenvectors e_0^s of the unperturbed generator A_0 , being averaged over properly selected time window, expose the beating phenomenon parametrized by the characteristics of the splitting starlet and the eigenbasis rotation:

$$\frac{1}{\Delta} \int_{T-\Delta/2}^{T+\Delta/2} E_0(P_s u(t)) dt \approx \frac{p_0^2}{2} \sum_{n,m} \bar{A}_\delta^n A_\delta^m \cos[\delta(\alpha_n - \alpha_m)t + \varphi_n - \varphi_m] \overline{\langle e_s^0, e_n^\delta \rangle} \langle e_s^0, e_m^\delta \rangle,$$

which looks quasi-random while the difference frequencies $\delta(\alpha_n - \alpha_m) \equiv \omega_m - \omega_n$ are incommensurable. In the case when there are only splitting of the multiplicity two $p_0 \rightarrow p_0 \pm \delta$ the beating is periodic, with the difference frequency $\delta\pi$, thus allowing to preview arising

maximal energy values of the migrating energy on the active zone, as noticed in the end of section I. In the case of incommensurable difference frequencies a similar previewing remains possible too, despite a quasi-random character of the beatings dynamics. The case of the tectonic plates is reduced to the above case of the connected oscillators via considering the boundary values of the solutions of the corresponding biharmonic wave equation $\rho u_{tt} + Lu = 0$. The corresponding spectral problem requires considering the boundary form for the generator $L : \langle Lu, v \rangle - \langle u, Lv \rangle$ which is reduced, see (Pavlov 2001) to a boundary integral $\int [\langle D\bar{u}, Nv \rangle - \langle N\bar{u}, Dv \rangle] d\Gamma$ and vanishes if appropriate boundary conditions with an hermitian matrix B are imposed: $[Nu - BDu]|_\Gamma = 0$ onto the boundary values Nu, Du .

Essential simplification of the original spectral problem is obtained while the Dirichlet-to-Neumann map, see (Pavlov 2001), transforming the boundary values of the homogeneous problem $Lu = \lambda \rho u$ $Nu = \mathcal{DN}(\lambda)Du$ one to another is substituted by an appropriate finite-dimensional rational approximation

$$\mathcal{DN}(\lambda) \longrightarrow P_E \mathcal{DN}(\lambda) P_E \equiv \mathcal{DN}_E(\lambda),$$

which corresponds to substitution of the original problem by a corresponding fitted solvable model (Pavlov et al 2010).

VI. APPENDIX C: WAVE EQUATION AND SEPARATION OF THE VARIABLES

The viscosity of the liquid underlay is small for relatively slow movements, which correspond to the frequency 200 μHz and typical amplitudes of the SGO. Based on analysis of a thin plate floating on a liquid underlay, we eliminate the hydrodynamic variables obtaining, see (Chung and Fox 2009), after an appropriate renormalisation, the biharmonic wave equation for the tranverse waves on the thin plate in the following form, see (Heisin 1967, Landau and Lifshitz 1970):

$$\begin{aligned} \rho h v_{tt} + \beta v_t + D \Delta^2 v + Q \Delta v &= 0 \quad v \xrightarrow{v=e^{i\omega t}u} \\ \longrightarrow D \Delta^2 u + i\omega \beta u + Q \Delta u &= \omega^2 \rho h u, \end{aligned} \quad (3)$$

Hereafter we neglect the liquid friction βu_t . It may be eliminated via an exponential factor $u \rightarrow \exp(-\beta t/2)u$ and a re-normalization of the frequency $\omega^2 \rightarrow \omega^2 - \frac{\beta^2}{4}$, while $\omega^2 - \frac{\beta^2}{4} > 0$.

The dependence of the frequency of the plane wave on the wave vector is given by

$$\omega = 2\pi \nu = \sqrt{\frac{Dk^4}{\rho h} - \frac{Qk^2}{\rho h}}. \quad (4)$$

For a spherical plate the separation of the variables is possible if the constant pressure force Q is applied in a spherically symmetric way. The wave equation can be factorized as

$$\begin{aligned} & \left(\sqrt{D}\Delta + \frac{Q}{2\sqrt{D}} + \sqrt{\omega^2 \rho h + \frac{Q^2}{4D}} \right) \times \\ & \times \left(-\sqrt{D}\Delta - \frac{Q}{2\sqrt{D}} + \sqrt{\omega^2 \rho h + \frac{Q^2}{4D}} \right) u = 0, \end{aligned} \quad (5)$$

and thus reduced to a pair of separate equations. Their solutions in the subspace E of functions, independent on the angular variable, represented on the spherical plate $0 \leq r \leq L$ via Bessel functions

$$\begin{aligned} & J_0 \left(\left[\frac{\omega \sqrt{\rho h}}{\sqrt{D}} \right]^{1/2} \left[\sqrt{1 + \frac{Q^2}{4\omega^2 \rho h D}} + \frac{Q}{2\omega \sqrt{\rho h} \sqrt{D}} \right]^{1/2} r \right), \\ & I_0 \left(\left[\frac{\omega \sqrt{\rho h}}{\sqrt{D}} \right]^{1/2} \left[\sqrt{1 + \frac{Q^2}{4\omega^2 \rho h D}} - \frac{Q}{2\omega \sqrt{\rho h} \sqrt{D}} \right]^{1/2} r \right), \end{aligned}$$

where J_0 is the standard Bessel function, and I_0 is a modified Bessel function of an imaginary argument $I_0(z) = J_0(iz)$. These solutions are regular at $r = 0$. However, large plate may be modelled by a circular plate with a circular hole in the centre. In this case we should add two other solutions, the Hankel functions H_0^1 and the modified Hankel function K_0 . One also may use a model where a small plate is a sector inside a circular large plate. Solutions on the large plate in this case are Bessel function with the index p determined by the angular size of the missing sector $\phi = \pi/p$, $p < 1$.

The eigenfunctions of the above biharmonic spectral problem on the small plate are obtained as a linear combinations of the Bessel functions J and I , and the eigenvalues are calculated, depending on the contracting tension Q , by the substituting of the linear combination into relevant boundary conditions on the border $r = L$ of the plate. Comparison of the eigenvalues with eigenfrequencies of the large plate defines the condition of resonances.

We postpone all relevant mathematical details to the oncoming publication by V. Flambaum, G.Martin and B. Pavlov “On the resonance interaction of seismogravitational modes on tectonic plates” (in preparation)

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